ON THE GROUPS OF COBORDISM Ω^k

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Introduction

In the papers [11] and [18] Rohlin and Thom have introduced an equivalence relation into the set of compact orientable (not necessarily connected) differentiable manifolds, which, roughly speaking, is described in the following manner: two differentiable manifolds are equivalent (*cobordantes*), when they together form the boundary of a bounded differentiable manifold. The equivalence classes can be added and multiplied in a natural way and form a graded algebra Ω relative to the addition, the multiplication and the dimension of manifolds. The precise structures of the groups of cobordism Ω^k of dimension k are not known thoroughly. Thom [18] has determined the free part of Ω and also calculated explicitly Ω^k for $0 \leqq k \leqq 7$.

The purpose of the present paper is to determine explicitly the groups Ω^k for $8 \leqq k \leqq 12$. Our method is analogous to that of Thom [18] and we shall calculate Ω^k using Serre's C-theory.

In § 1 we explain shortly some general results on the Eilenberg-MacLane complexes, Serre's C-theory and the Grassmann manifold, which will be used later. In § 2 the homotopy groups of the Thom complex $M(SO(n))$ associated with the rotation group are calculated. In § 3 we determine the groups of cobordism Ω^k for $8 \leqq k \leqq 12$, and discuss some problems related to Ω^k.

Some of the results contained in this paper have been announced in the note [1].

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§ 1. Preliminaries

Before we approach the determination of the homotopy groups of the Thom complex $M(SO(n))$ associated with the rotation group, it is necessary to recall

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