ON THE EXISTENCE OF UNRAMIFIED SEPARABLE INFINITE SOLVABLE EXTENSIONS OF FUNCTION FIELDS OVER FINITE FIELDS*

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In the present note, using the results in the previous paper,[1] we shall prove the following existence theorem:

THEOREM. *Let k be a finite field with q elements and K/k be a regular extension of dimension one over k. Then, if $q \geqq 11$ and the genus g_K of K/k is greater than one, there exists an unramified separable infinite solvable extension of K which is regular over k.*[2]

§ 1. The results in [1]

1.1. Let k be a finite field with q elements and K/k be a regular extension of dimension one over k. Let L/k be an unramified separable normal extension of K which is also regular over k. We denote by $G(L/K)$ the galois group of L/K. We denote by C_L and C_K non-singular complete models of K/k and L/k, respectively, and denote by $\hat{\pi}_{L/K}$ the trace mapping of C_L onto C_K. We denote by $J_L(J_K)$ and $\varphi_L(\varphi_K)$ the jacobian variety of $C_L(C_K)$ and a canonical mapping of $C_L(C_K)$ into $J_L(J_K)$, respectively, where we may assume that $J_L(J_K)$ and $\varphi_L(\varphi_K)$ are also defined over k. We denote by $\pi_{L/K}$ the extension of $\hat{\pi}_{L/K}$ which is a homomorphism of J_L onto J_K such that $\pi_{L/K} \circ \varphi_L = \varphi_K \circ \hat{\pi}_{L/K} + c$ with a constant point c. After a suitable translation of φ_K, we assume that

$$\pi_{L/K} \circ \varphi_L = \varphi_K \circ \hat{\pi}_{L/K}. \tag{1}$$

We denote by $A(\quad, k)$ the subgroup of k-rational points of a commutative group variety A.

Each element ε_ν of $G(L/K)$ induces an automorphism $\{\eta_L(\varepsilon_\nu)\}$ of J_L and

Received October 25, 1957.

* This note was prepared while the author was a Yukawa Fellow at Osaka University.

[1] We shall refer this paper with [1].

[2] We mean by an infinite solvable extension a solvable extension of infinite degree.