AN INVESTIGATION ON THE LOGICAL STRUCTURE OF MATHEMATICS (III)[0)]

FUNDAMENTAL DEDUCTIONS

SIGEKATU KURODA

In this Part (III) the proofs in UL are given for fundamental formulas concerning the following dependent variables:

Elementary set	$\{a_1, \ldots, a_n\}$;
Ordered pair[1)]	$\langle a, b\rangle$;
Image[2)] by σ of a	$\sigma`a$;
Image[2)] by σ of elements of a	$\sigma``a$;
Domain of operator	D_σ;
Range of operator	W_σ;
Uniqueness	Un;
Bi-uniqueness	Un_2;
Inverse operator	σ^{-1};
One-to-one mapping	$\mathrm{Map}_2^{a,b}$;
Composition of operators	$\sigma\circ\tau$;
Restriction of operator	$\sigma\restriction a$;
Identical mapping	ι.

The following defining formulas are used for these dependent variables:

$$u\in\{a_1, \ldots, a_n\}\equiv u=a_1\vee\ldots\vee u=a_n,$$

$$u\in\langle ab\rangle\equiv u=\{a\}\vee u=\{ab\},$$

$$u\in\sigma`a\equiv\exists x.\ \langle ax\rangle\in\sigma\wedge u\in x,$$

$$u\in\sigma``a\equiv\exists x.\ x\in a\wedge\langle xu\rangle\in\sigma,$$

$$u\in\mathrm{D}_\sigma\equiv\exists x.\ \langle ux\rangle\in\sigma,$$

$$u\in\mathrm{W}_\sigma\equiv\exists x.\ \langle xu\rangle\in\sigma,$$

$$u\in\mathrm{Un}\equiv\forall xyz.\ \langle xy\rangle\in u\wedge\langle xz\rangle\in u\rightarrow y=z,$$

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0) See foot note 0) in Part (IV) published in this same volume.

1) $\langle a, b\rangle$ is also written as $\langle ab\rangle$.

2) $\sigma`a$ as well $\sigma``a$ are also written in the same way as a^σ, when the distinction is clear by the context.