NOTE ON THE GROUP OF AFFINE TRANSFORMATIONS OF AN AFFINELY CONNECTED MANIFOLD

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The purpose of the present note is to reform Mr. K. Nomizu's result[1] on the group of all affine transformations of an affinely connected manifold. We shall prove the following.

THEOREM. *The group of all affine transformations of an affinely connected manifold is a Lie group.*

Mr. K. Nomizu proves the theorem when the affinely connected manifold is complete.[2] And he gives out a question whether this assumption of completeness is really necessary. We shall show it is possible to prove the theorem without any assumption by considering a Riemannian metric in the bundle of frames of the manifold, which is naturally defined by the affine connection.

After preparing this note we heard from Mr. Nomizu that he has also proved the same theorem and using this result Mr. S. Kobayashi[3] has proved similar results on transformation groups of manifolds with certain connections.

In section 1 we resume the definitions and properties about affine connections, geodesic curves and regular neighbourhoods, which are given in Mr. K. Nomizu's paper. The definition of the group of affine transformations is given in section 2. The proof of the theorem is given in the last four sections.

1. Let M be a connected differentiable manifold[4] of dimension n with an

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1) K. Nomizu; On the group of affine transformations of an affinely connected manifold, Proc. Amer. Math. Soc. vol. 4 (1953).

2) For the definition of "completeness" see.[1]

3) S. Kobayasi; Groupe de transformations qui laissent invariante une connexion infinitesimale, Comptes rendus, 238 (1954).

4) The term "differentiable" will always mean "of class C^∞". As for the definitions and the notations of manifold, tangent vector, differential form, etc. we follow C. Chevalley; Theory of Lie groups, Princeton University Press, 1946. A manifold is not necessarily connected.