ON THE DIMENSION OF PARACOMPACT HAUSDORFF SPACES

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This short note gives the generalized sum theorem for Lebesgue dimension of paracompact Hausdorff spaces. Our theorem, though it is a generalization of Mr. Morita's sum theorem for fully normal spaces [3, Theorem 3. 2] which is essentially based on his generalized sum theorem for normal spaces [3, Theorem 3. 1], is obtained by very brief arguments, using only the usual sum theorem for normal spaces.

THEOREM 1. *A paracompact Hausdorff space R is $\leqq n$-dimensional if and only if it is locally $\leqq n$-dimensional.*[1)]

Proof. 'Only if' part is obvious. We shall prove 'if' part. For every point $p \in R$, there exists an open neighbourhood $V(p)$ whose closure is $\leqq n$-dimensional. Since R is strongly screenable [4, Theorem 2], there exists an open covering $\{U_\alpha ; \alpha \in \bigcup_{i=1}^{\infty} A_i\}$ which refines $\{V(p) ; p \in R\}$ such that each $\mathfrak{U}_i = \{U_\alpha ; \alpha \in A_i\}$ forms a discrete collection. It can easily be seen, from $\leqq n$-dimensionality of $\overline{U}_\alpha$ and from discreteness of $\mathfrak{U}_i$, that each $\overline{U}_i = \bigcup_{\alpha \in A_i} \overline{U}_\alpha$ is $\leqq n$-dimensional. Thus R is $\leqq n$-dimensional as the sum of countable $\leqq n$-dimensional closed sets [1, Theorem 4. 2]. Q.E.D.

THEOREM 2. (Generalized sum theorem) *If a paracompact Hausdarff space R is covered by a collection of $\leqq n$-dimensional closed sets, $\{F_\alpha ; \alpha \in A\}$, which is locally countable,*[2)] *R is $\leqq n$-dimensional.*

Proof. Let p be an arbitrary point of R and $V(p)$ be a neighbourhood of it such that $A(p) = \{\alpha ; \overline{V(p)} \cap F_\alpha \neq \phi\}$ consists of at most countable sets of indices. Since R is normal, $\overline{V(p)}$ is normal as a relative space. Then the sum

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1) A space is called locally $\leqq n$-dimensional, if every point of the space has a closed neighbourhood whose dimension is at most n.

2) A collection of subsets of R is called locally countable, if every point of R has an open neighbourhood of it which meets at most countable elements of the collection.