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SATURATED IDEALS IN BOOLEAN EXTENSIONS

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0. Introduction. Let κ be an uncountable cardinal, and let λ be a regular cardinal less than κ. Let I be a λ-saturated non-trivial ideal on κ. Prikry, in his thesis, showed that, in certain Boolean extensions, κ has a λ-saturated non-trivial ideal on κ. More precisely,

THEOREM (Prikry [8]). *Let κ, λ and I be as above. Let $\mathscr{B}$ be a λ-saturated complete Bollean algebra. Let $J \in V^{(\mathscr{B})}$ such that, with probability* **1**, *J is the ideal on $\check{\kappa}$ generated by $\check{I}$. Then, it is $\mathscr{B}$-valid that J is a $\check{\lambda}$-saturated non-trivial ideal on $\check{\kappa}$.*

The following questions naturally arise; 1) If I is κ-saturated (κ^+-saturated), does J remain κ-saturated (κ^+-saturated)? 2) If $\mathrm{sat}(\mathscr{B}) = \kappa$, what is the saturatedness of J?

For 1), we obtain the following theorem.

THEOREM 1. *Let κ and λ be as above. Let γ be a regular cardinal such that $\lambda \leq \gamma \leq \kappa^+$, and let I be a γ-saturated non-trivial ideal on κ. Let $\mathscr{B}$ be a λ-saturated complete Boolean algebra. Then, it is $\mathscr{B}$-valid that J is γ-saturated.*

For 2), we get the following theorems.

THEOREM 2. *Let κ be an uncountable cardinal, and I be a κ-saturated non-trivial ideal on κ. Let $\mathscr{B}$ be a homogeneous complete Boolean algebra such that $\mathrm{sat}(\mathscr{B}) = \kappa$. Then, it is $\mathscr{B}$-valid that J is not κ-saturated.*

THEOREM 3. *Let κ be a measurable cardinal, and I be a non-trivial prime ideal on κ. Let $\mathscr{B}$ be a homogeneous complete Boolean algebra such tnat $\mathrm{sat}(\mathscr{B}) = \kappa$. Then, it is $\mathscr{B}$-valid that J is not κ^+-saturated.*

We will prove the above theorems as applications of a certain useful lemma, which will be proved in § 4.

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