N. C. Jain and G. Kallianpur
Nagoya Math. J.
Vol. 47 (1972), 15–28

OSCILLATION FUNCTION OF A MULTIPARAMETER GAUSSIAN PROCESS

NARESH C. JAIN* AND G. KALLIANPUR*

1. Introduction. It is our object in this paper to show that the recent results of K. Ito and M. Nisio [4] on the oscillation function of Gaussian processes on $[0,1]$ are valid for Gaussian processes with a general multiparameter "time" set T. Except in extending Theorem 4 of [4] where we assume T to be the d-dimensional cube, in all other cases we allow T to be a separable metric space. Despite the generality of the time set, the proofs are achieved essentially using the method of the above mentioned authors. However, in Theorem 1 below we find the use of Lemma 6 of [5] more convenient than the approach via orthogonal expansions and Kolmogorov's zero-one law as is done in [4].

The extension of Ito and Nisio's results has been undertaken with a view to considerably enlarging their scope of application. One application is Theorem 5 which states that for certain multi-parameter (S,Γ) stationary Gaussian processes (see Section 5 for the definition) the oscillation function is either identically 0 or identically ∞ and consequently that Yu. K. Belyaev's 0–1 law [1] holds for these processes. As a corollary we show that the same conclusion holds for Gaussian stationary processes (often called homogeneous random fields) given on homogeneous spaces. The corollary to Theorem 5 has also been obtained by Eaves [2] using a method which seems to be based on Belyaev's original proof. Theorems 1–3 of [4] have been extended to the case where $T=[0,1]^d$ (though not to more general parameter sets) by Kawada [6].

Let T be a separable metric space and let Ω be the space of all real-valued functions T. Let $\mathscr{B}$ denote σ-algebra generated by the cylinder sets. Let P be a Gaussian probability measure on $(\Omega,\mathscr{B})$ such that the random

Received July 27, 1971.
Revised October 14, 1971.
* This work was partially supported by an NSF Grant.