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NORMAL LIGHT INTERIOR FUNCTIONS DEFINED IN THE UNIT DISK

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1. Preliminaries

Let D be the unit disk, C the unit circle, and f a continuous function from D into the Riemann sphere W. We say that f is *normal* if f is uniformly continuous with respect to the non-Euclidean hyperbolic metric in D and the chordal metric in W. Let $\chi(w_1, w_2)$ denote the chordal distance between the points $w_1, w_2 \in W$; and let $\rho(z_1, z_2)$ denote the non-Euclidean hyperbolic distance between the points $z_1, z_2 \in \in D$ [6]. If $\{z_n\}$ and $\{z'_n\}$ are two sequences of points in D with $\rho(z_n, z'_n) \to 0$, we say that $\{z_n\}$ and $\{z'_n\}$ are *close sequences.*

Let A be an open subarc of C, possibly C itself. A *Koebe sequence of arcs relative to* A is a sequence $\{J_n\}$ of Jordan arcs such that: (a) for every $\varepsilon > 0$,

$$J_n \subset \{z \in D : |z - a| < \varepsilon \text{ for some } a \in A\}$$

for all but finitely many n, and (b) every open sector $\varDelta$ of D subtending an arc of C that lies strictly interior to A has the property that, for all but finitely many n, the arc J_n contains a subarc L_n lying wholly in $\varDelta$ except for its two end points which lie on distinct sides of $\varDelta$.

We say that the function f has the limit c along the sequence of arcs $\{J_n\}$ (denoted by $f(J_n) \to c$) provided that, for every $\varepsilon > 0$, $\chi(c, f(J_n)) < \varepsilon$ for all but finitely many n.

2. Factorization of light interior functions

Let f be a light interior function from D into W, i.e. f is an open map which does not take any continuum into a single point. Church [4, p. 86] has pointed out that f has the representation $f = g \circ h$ where h is a

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