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ON A CLASS OF METRICAL AUTOMORPHISMS ON GAUSSIAN MEASURE SPACE

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To Professor KATUZI ONO on the occasion of his 60th birthday

1. Introduction. Let E be an infinite dimensional real nuclear space and H be its completion by a continuous Hilbertian norm $\| \ \|$ of E. Then we have the relation

$$E \subset H \subset E^*$$

where E^* is the conjugate space of E. Consider a function $C(\xi)$ on E defined by the formula

$$C(\xi) = e^{-\|\xi\|^2/2}, \quad \xi \in E. \tag{1}$$

Then $C(\xi)$ is a positive definite and continuous function with $C(0)=1$. Therefore, by Bochner-Minlos' theorem, there exists a unique probability measure μ on E^* such that

$$\int_{E^*} e^{i\langle x, \xi\rangle}\, d\mu(x) = e^{-\|\xi\|^2/2}, \quad \xi \in E, \tag{2}$$

where $\langle x, \xi\rangle$ being the canonical bilinear form. The measure μ is defined on the σ-algebra $\mathscr{L}$ generated by all cylinder sets of E^* ([1]). We call μ a *Gaussian measure.*

Let $O(H)$ be the group formed by all linear and orthogonal operators acting on H. After [3], we consider a subgroup $O(E)$ of the group $O(H)$ which is defined as the collection of all $g's$ of $O(H)$ having the property that each of g is a linear homeomorphism from E onto E. An operator g of $O(E)$ is called a rotation of E. As is seen from the formula (2), the conjugate operator g^* of a rotation g is a metrical automorphism on the space (E^*, μ). The purpose of this paper is to generalize this fact and we shall prove that

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