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ON A CLASSICAL THETA-FUNCTION

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To Professor Katuzi Ono on his 60th birthday

The purpose of this paper is to get a certain explicit expression of automorphic factors, formulated rather differently than usual, of the classically well known theta function[1])

$$\vartheta(z) = \vartheta_3(0, z) = \sum_{m=-\infty}^{\infty} e^{\pi i m^2 z}, \quad (z = x + iy, \ y > 0). \tag{1}$$

The special linear group $G = SL(2, \boldsymbol{R})$ over the real field $\boldsymbol{R}$ has a 2-fold topological covering group $\tilde{G}$, and the maximal compact subgroup $T = SO(2)$ of G has also a naturally corresponding 2-fold covering group $\tilde{T}$ in $\tilde{G}$. While the upper half plane H is usually identified with the homogeneous space G/T, the properties discussed in §1 of the automorphic factors of $\vartheta(z)$, (13) among others, show directly that for the purpose of investigating $\vartheta(z)$ it is legitimate to identify the upper half plane H with $\tilde{G}/\tilde{T}$. Moreover, as we see in §2, the quadratic reciprocity law in the rational number field $\boldsymbol{Q}$ can be formulated as a multiplicativity of a number-theoretical function defined on a discrete subgroup of $\tilde{G}$. For a totally imaginalry number field this kind of result was already stated in [4] in a simpler form, but in general we need the covering group $\tilde{G}$.

It is famous in number theory that there is a close relationship between the quadratic reciprocity law and the function $\vartheta(z)$[2]). The investigation in this paper, inclusive of all explicit calculations, may be regarded as a trial to catch as simply as possible the theoretical background of that interesting phenomenon.

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1) Called in many cases theta constant. It is an automorphic form with respect to the discontinuous group Γ defined in §1.

2) For example, see [2].