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GREEN'S FUNCTIONS FOR GENERALIZED SCHROEDINGER EQUATIONS*

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I. Introduction. The purpose of this paper is to discuss functions defined on the continuous sample paths of Gaussian Markov processes which serve as Green's functions for pairs of generalized Schroedinger equations. The results extend the author's earlier paper [2] to a forward time version, and consider different boundary conditions.

The approach is similar to another paper, "Feynman-Cameron Integrals," [4]. The restrictions on the potential $V[y, t]$ needed to prove existence theorems are so strict as to rule out most practical potentials. Hence, the first few theorems are designed to lead the reader to an expression which in many cases can be shown to satisfy the equations and boundary conditions.

A sequential approach is used for theoretical reasons, and because it would lead naturally into finite-dimensional approximations similar to those of [4]. It also may serve as background for numerical analysis work similar to that done for the conditional Wiener integral by L.D. Fosdick, [9]. It also reveals interesting facts about the conditional Wiener and Gaussian Markov processes. In particular, we will calculate the mean function for the conditioned Gaussian Markov process, and the covariance function for the conditioned Wiener process.

The wave function for the forced harmonic oscillator is calculated. This is one of the few time-dependent potentials considered in the numerous function space integral papers on this subject. It should be mentioned that Feynman and Hibbs [8] consider this example in a different manner. Itô [10] considered the regular harmonic oscillator. Donsker and Lions considered time-dependent potentials in [7].

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