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ON THE UPPER AND LOWER CLASS FOR GAUSSIAN PROCESSES WITH SEVERAL PARAMETERS

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1. In the study on Hölder-continuity of Brownian motion, A.N.Kolmogorov introduced the concept of upper and lower classes and presented a criterion with the integral form to test whether some function belongs to upper or lower class; the so-called Kolmogorov's test (I.Petrovesky [10]). P.Lévy considered the upper and lower class with regard to the uniform continuity of Brownian motion. We shall recall the definition of the upper and lower classes. We shall call $\varphi(t)$ a function belonging to the upper class with regard to the uniform continuity of Brownian motion $x(t)$ if there exists a positive number $\varepsilon(w)$ such that, for almost all w,

$$|t-t'| \leqq \varepsilon(w) \qquad \text{implies}$$

$$|x(t)-x(t')| \leqq |t-t'|^{1/2} \cdot \varphi(1/|t-t'|). \tag{1.1}$$

On the otherhand, we shall call $\varphi(t)$ a function belonging to the lower class with regard to the uniform continuity of Brownian motion $x(t)$ if, for almost all w and for any positive number δ, there exist a pair (t, t') such that $|t-t'| \leqq \delta$ and (1.1) does not hold.*)

P.Lévy showed that the function

$$\varphi(t) = c \cdot (2 \log t)^{1/2}$$

belongs to the upper class if $c > 1$ and to the lower class if $c < 1$ (P. Lévy [8]). Further, K.L.Chung, P.Erdös and T.Sirao [3] proved that a continuous, non-negative and non-decreasing function $\varphi(t)$ belongs to upper or lower class according as the integral

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*) It turns out that every continuous positive and non-decreasing function belongs to either upper class or lower class.