B.G. Basmaji
Nagoya Math. J.
Vol. 35 (1969), 99–107

MONOMIAL REPRESENTATIONS AND METABELIAN GROUPS[1)]

B.G. BASMAJI

1. Introduction. In this paper we develop a method to find all the irreducible and inequivalent representations over the complex field C of a family of finite groups that includes the metabelian groups. The outline of the paper is as follows: In §2 we let P be a one-dimensional representation of a subgroup H of a finite group G and find a maximal subgroup K, $H \subseteq K \subseteq G$, such that an extension $\bar{P}$ of P to K exists. We show that the induced representation $\bar{P}^G$ is irreducible if K is normal in G. In §3 we give all the irreducible and inequivalent representations of the "generalized metabelian group" G, and in particular of the metabelian group, and decompose the group ring CG into its simple components. The convenience of this method is shown in §4 where we determine the representations of the metacyclic group and a metabelian group of order $2sp^2$, p an odd prime and $s \mid p-1$. The algorithm in §5 is supplementary and can be used to find representations of more general groups than the two in §4.

In this paper we make use of theorems 45. 2 and 45. 6 of Curtis-Reiner [3, §45], in the special case (due to Shoda [6]) where L, L_1, L_2 afford one-dimensional representations.

2. Induced representations. Let G be a finite group and H, D, and K be subgroups of G such that $G \supseteq K \supseteq H \supseteq D$ with the following conditions:

(i) D is normal in H and H/D is cyclic.

(ii) $K' \cap H \subseteq D$ where K' is the commutator group of K.

Received June 12, 1968

1) This paper is essentially Chapter III of the author's dissertation submitted to the University of Maryland for the Ph.D. degree. The author wishes to thank Professor Kuroda for his advice. Also, this research was partially supported by NSF grants GP–1837 and GP–6010.