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SEPARABLE EXTENSIONS AND CENTRALIZERS OF RINGS

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We have introduced in [9] a type of separable extensions of a ring as a generalization of the notion of central separable algebras. Unfortunately it was unsuitable to call such extensions 'central' as Sugano pointed out in [15] (Example below Theorem 1. 1). Some additional properties of such extensions were given in [15]. Especially Propositions 1. 3 and 1. 4 in [15] are interesting and suggested us to consider the commutor theory of separable extensions. Let Λ be a ring and Γ a subring of Λ. When $\Lambda \otimes_\Gamma \Lambda$ is a direct summand of a finite direct sum of Λ as a two-sided Λ-module we shall denote it by ${}_\Lambda\Lambda \otimes_\Gamma \Lambda_\Lambda < \oplus\, {}_\Lambda(\Lambda \oplus \cdots \oplus \Lambda)_\Lambda$ and call Λ an H-separable extension of Γ (cf. [9] and [15]). Let Δ be a subring of Λ containing the center C of Λ and let Γ be the centralizer of Δ in $\Lambda, \Gamma = V_\Lambda(\Delta) = \Lambda^\Delta = \{\lambda \in \Lambda \mid \delta\lambda = \lambda\delta,\ \delta \in \Delta\}$. If ${}_\Lambda\Lambda \otimes_C \Delta_\Delta < \oplus\, {}_\Lambda(\Lambda \oplus \cdots \oplus \Lambda)_\Delta$ and Δ is C-finitely generated and projective then Λ is an H-separable extension of Γ and Λ is right Γ-finitely generated and projective. Conversely for such an H-separable extension Λ over Γ, if we set $\Delta' = V_\Lambda(\Gamma)$, then ${}_\Lambda\Lambda \otimes_C \Delta'_{\Delta'} < \oplus\, {}_\Lambda(\Lambda \oplus \cdots \oplus \Lambda)_{\Delta'}$ and Δ' is C-finitely generated and projective. In this way we can give a one to one correspondence between Γ's and Δ's. A more general situation than H-separable extensions is possible and is symmetric to each other. Let B and Γ be subrings of Λ such that $B \supset \Gamma$. Let $\Delta = V_\Lambda(\Gamma)$ and $D = V_\Lambda(B)$. If ${}_BB \otimes_\Gamma \Lambda_\Lambda < \oplus\, {}_B(\Lambda \oplus \cdots \oplus \Lambda)_\Lambda$ and B is right Γ-finitely generated and projective then ${}_\Lambda\Lambda \otimes_D \Delta_\Delta < \oplus_\Lambda(\Lambda \oplus \cdots \oplus \Lambda)_\Delta$ and Δ is left D-finitly generated and projective. Same considerations are possible for H-separable subextensions. These are treated in §2, 3 and 4. §1 is a continuation of §1 in [9] and the results are applied to the following sections. In §5 we give some notes on two-sided modules. It is well known that any finitly generated projective module over a commutative ring is a generator (completely faithful) if it is faithful. Let M be a two-sided module over a

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