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Nagoya Math. J.
Vol. 33 (1968), 53-56

A NOTE ON TANGENTIAL EQUIVALENCES

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The main objective of this paper is to prove the following theorem, which generalizes some results of [1], [2], [6]. Our theorem is also suggested by the work of Novikov [5].

THEOREM. *Let M, M' be closed smooth $2n$-manifolds of the same homotopy type. Let $\tau(M)$ and $\tau(M')$ be the tangent bundles of M and M'. Suppose we are given a homotopy equivalence $f: M \to M'$ such that the induced bundle $f^*\tau(M')$ is stably equivalent to $\tau(M)$. (cf. [4]). Then $f^*\tau(M')$ is actually equivalent to $\tau(M)$.*

COROLLARY. *Under the same assumption, M and M' have the same span, that is the maximal numbers of linearly independent vector fields on M and M' are equal. (cf. [1]).*

Proof of the theorem. Let M^{2n-1} be the $(2n-1)$-skeleton of M. Set $\tau = \tau(M)$ and $\tau' = f^*\tau(M')$. Let $\tau|M^{2n-1}$ and $\tau'|M^{2n-1}$ be the restrictions of τ and τ' on M^{2n-1}. Let $O(k)$ be the orthogonal group of the k-dimensional euclidean space R^k. Then $(O(2n+1), O(2n))$ is $(2n-1)$-connected. By our assumption $\tau|M^{2n-1}$ is equivalent to $\tau'|M^{2n-1}$, and using the obstruction theory we have an equivalence $\alpha: \tau|M^{2n-1} \approxeq \tau'|M^{2n-1}$ which can be extended to a stable equivalence of $\tau \oplus 1 \approxeq \tau' \oplus 1$ over M, where 1 is the trivial line bundle over M.

Let $i: O(2n) \to O(2n+1)$ be the canonical inclusion. Then we have the following exact sequence

$$O \to \text{Ker } i_* \xrightarrow{j} \pi_{2n-1}(O(2n)) \xrightarrow{i*} \pi_{2n-1}(O(2n+1)) \to O,$$

where Ker $i_* \approx Z$ (the additive group of integers) (cf. [3]). Let c be the obstruction cocycle for extending α to an equivalence $\tau \approxeq \tau'$ over the whole M. (The coefficients group $\pi_{2n-1}(O(2n))$ of this cocycle is twisted if M is non-orientable, and the operation of $\pi_1(M)$ is given in [7] § 23). Then, by our previous remark on α, the value $c(\sigma_i^{2n})$ of c on each simplex σ_i^{2n} of M

Received January 18, 1968.