APPROXIMATION OF UNIFORM TRANSPORT PROCESS ON A FINITE INTERVAL TO BROWNIAN MOTION

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§ 1. Introduction.

Let us consider a finite closed interval $[-a, a]$ which will be thought of as being a medium capable of transporting particles. These particles may move only to the right or to the left with the constant speed c, and each particle changes the moving-direction during the time Δ with probability $k\Delta + o(\Delta)$. If a right- (left-) moving particle hits the boundary point a $(-a)$, then either it turns to the left (right) with probability $1-q_1(1-q_{-1})$ or dies with probability $q_1(q_{-1})$. The particle, changed the moving-direction, starts afresh from that position. Now, let $x(t)$ be the coordinate of the particle at time t and let $\theta(t)$ be 1 or -1 according as the moving-direction at time t is right or left. Then $\mathbf{X}(t) = (x(t),\ \theta(t))$ can be considered as a Markov process over the state space $\boldsymbol{S} = \{(x, \theta)\ -a \leqq x \leqq a,\ \theta = \pm 1\}$. For short, we shall call it *a uniform transport process* over the interval $[-a, a]$. we shall give the precise definition in § 2.

Let $\boldsymbol{T}_t$ be the semigroup corresponding to the Markov process $\boldsymbol{X}(t) = (x(t),\ \theta(t))$. Then $u(t, x, \theta) = \boldsymbol{T}_t f(x, \theta)$ will be the solution of the following differential equation:

$$(\boldsymbol{T}) \quad \begin{cases} \dfrac{\partial}{\partial t} u(t, x, 1) = c\dfrac{\partial}{\partial x} u(t, x, 1) - ku(t, x, 1) + ku(t, x, -1) \\ \dfrac{\partial}{\partial t} u(t, x, -1) = -c\dfrac{\partial}{\partial x} u(t, x, -1) - ku(t, x, -1) + ku(t, x, 1) \\ u(t, a, 1) = (1-q_1)u(t, a, -1) \\ u(t, -a, -1) = (1-q_{-1})u(t, a, 1) \\ u(t, x, \theta) \to f(x, \theta) \ \text{as} \ t \to 0 \end{cases}$$

At this stage, choose constants c, k, q_1, q_{-1} so that

$$(\boldsymbol{C}_1) \qquad c^2/k = 1$$

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