# A NOTE ON TRIPLE SYSTEMS AND TOTALLY GEODESIC SUBMANIFOLDS IN A HOMOGENEOUS SPACE

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**1. Introduction.** In the study of nonassociative algebras various "triple systems" frequently arise from the associator function and other multilinear objects. In particular Lie triple systems arise in the study of Jordan algebras and a generalization of a Lie triple system arises in Malcev algebras. Lie triple systems also are used to study totally geodesic submanifolds of a Riemannian symmetric space. We shall show how a generalization of Lie triple systems also arises from the study of curvature and geodesics of a torsion free connexion on a manifold and bring out the relation of this to various nonassociative algebras.

To establish notation we briefly review connexions, curvature and the second fundamental form in sections 2 and 3 and explicitly indicate how the results of [13] can be generalized to a manifold and to the study of the second fundamental form. In section 4 we use the results of [6] on reductive homogeneous spaces to show the relationships between certain nonassociative algebras, generalization of Lie triple systems and totally geodesic submanifolds. In section 5 we shall show that the Lie algebra of the holonomy group and the curvature formula for a reductive homogeneous space are well behaved in terms of certain nonassociative algebras. Also we shall give examples for which the curvature formula and the totally geodesic submanifolds are easy to determine by these algebras.

**2. Basics.** To establish notation we review some basic facts about covariant differentiation, torsion and curvature as given in [1, 3]. Let $M$ be a $C^\infty$ manifold and let $D$ be a covariant differentiation operator (i.e. connexion) defined on $M$. Thus for each pair of $C^\infty$ vector fields defined on a suitable domain $A \subset M$ [3] we have a $C^\infty$ vector field $D_X Y = D(X, Y)$ with domain $A$ such that if $Z$ is a $C^\infty$ vector field on $A$ and $f$ a $C^\infty$ real valued function on $A$, then $D$ satisfies

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