ON OSCULATING SYSTEMS OF DIFFERENTIAL EQUATIONS OF TYPE (N, 1, 2)

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The main subject in the present article has the origin in the following quite primitive question: *Linear systems of ordinary differential equations form a nice family. Then, from the projective point of view, what does correspond to linear systems?*

An osculating system of ordinary differential equations of type $(N, 1, 2)$ means a system of differential equations

$$(*) \quad \det \begin{pmatrix} y_{\alpha_0}, & y_{\alpha_1} \\ \frac{dy_{\alpha_0}}{du}, & \frac{dy_{\alpha_1}}{du} \end{pmatrix} = F_{\alpha_0, \alpha_1}(u, y_0, \ldots, y_N) \qquad (0 \leqslant \alpha_0 < \alpha_1 \leqslant N)$$

such that F_{α_0, α_1} $(0 \leqslant \alpha_0 < \alpha_1 \leqslant N)$ are quadratic forms in $y_0, \ldots, y_N$. If a vector $(\varphi_0, \ldots, \varphi_N)$ is a solution of (*), then for any holomorphic function ϕ the vector $(\phi\varphi_0, \ldots, \phi\varphi_N)$ is also a solution (*). Hence the map: $u \to (\varphi_0(u), \ldots, \varphi_N(u))$ into the projective N-space $\boldsymbol{P}_N$ has a nice meaning. We shall call such a map a projective solution of (*). From the projective point of view, roughly speaking, the system (*) is equivalent to the following systems

$$(**) \qquad \frac{d\frac{y_\alpha}{y_\beta}}{du} = F_{\alpha, \beta,}(u, y_0, \ldots, y_N) \qquad (0 \leqslant \alpha, \beta \leqslant N)$$

where $F_{\alpha,\beta} + F_{\beta,\alpha} = 0$ $(0 \leqslant \alpha, \beta \leqslant N)$. The initial variety $\boldsymbol{W}_{u_0}^{(F)}$ at a regular point u_0 for (*) means the set of all the point x in the projective N-space $\boldsymbol{P}_N$ such that there exists a holomorphic projective solution of (*) with the initial point x at $u = u_0$.

Then the following comparative table shows that osculating systems of type $(N, 1, 2)$ together with their projective solutions give an answer to our primitive question.

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