SPIRAL ASYMPTOTIC VALUES OF FUNCTIONS MEROMORPHIC IN THE UNIT DISK

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1. Introduction

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Alice Roth has made an extensive study of entire meromorphic functions with prescribed behavior along half rays emanating from the origin (6). The question arose whether analogous results could be found for functions meromorphic in the unit disk with the same behavior prescribed along an exhaustive class of spirals emanating from the origin. In this paper, I present a class of spirals which satisfactorily fills this role. However, I make no claim to the effect that only this class will suffice.

2. The main results

The circle with origin as center and radius k in the complex plane will be denoted by $C(k)$ throughout this paper. Similarly, the open disk with origin as center and radius k will be denoted by $D(k)$.

(2. 1) Definition. For each value θ, $0 \leq \theta < 2\pi$, define $S_\theta = \left\{ z = r \exp\left[i\left(\theta + \tan\frac{\pi r}{2}\right)\right], 0 \leq r < 1 \right\}$. Further, define $S_{-\theta} = S_{2\pi-\theta}$ for each θ, $0 \leq \theta \leq 2\pi$. These spirals will be called Study spirals (7, p. 45).

Notice that each Study spiral originates at the origin and its argument tends monotonically to $+\infty$ as r tends to 1. Also if $\theta_1 \neq \theta_2$, then S_{θ_1} and S_{θ_2} have only the origin in common.

(2. 2) Definition. Let $\mathscr{S}$ represent the class of functions meromorphic and non-constant in $D(1)$ and which tend to a definite limit, finite or infinite, on each Study spiral as r tends to 1. (Theorem 2.15 asserts that this class is rather large.) For each function $F(z)$ define the spiral limit value function $f(\theta)$, $0 \leq \theta < 2\pi$, associated with $F(z)$ as follows:

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