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REPRESENTATIONS OF QUADRATIC FORMS

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0. We have shown in [1]

THEOREM A. *Let L be a lattice in a regular quadratic space U over $\boldsymbol{Q}$; then L has a submodule M satisfying the following conditions* 1), 2):

1) $dM \neq 0$, $\operatorname{rank} M = \operatorname{rank} L - 1$, *and M is a direct summand of L as a module.*

2) *Let L' be a lattice in some regular quadratic space U' over $\boldsymbol{Q}$ satisfying $dL' = dL$, $\operatorname{rank} L' = \operatorname{rank} L$, $t_p(L') \geq t_p(L)$ for any prime p. If there is an isometry α from M into L' such that $\alpha(M)$ is a direct summand of L' as a module, then L' is isometric to L.*

Our aim is to remove such a restriction in 2) that $\alpha(M)$ is a direct summand of L' as a module:

THEOREM B. *Let L be a lattice in a regular quadratic space U over $\boldsymbol{Q}$; then L has a submodule M with* $\operatorname{rank} M = \operatorname{rank} L - 1$, $dM \neq 0$ *which is a direct summand of L as a module and satisfies*

(*) *let L' be a lattice in some regular quadratic space U' over $\boldsymbol{Q}$ satisfying $dL' = dL$, $\operatorname{rank} L' = \operatorname{rank} L$, $t_p(L') \geq t_p(L)$ for any prime p; if there is an isometry α from M into L', then L' is isometric to L.*

1. Notations and some lemmas

We denote by $\boldsymbol{Q}, \boldsymbol{Z}, \boldsymbol{Q}_p$ and $\boldsymbol{Z}_p$ the rational number field, the ring of rational integers, the p-adic completion of $\boldsymbol{Q}$, and the p-adic completion of $\boldsymbol{Z}$, respectively. For a quadratic space U we denote $Q(x), B(x, y)$ the quadratic form and the bilinear form associated with U $(2B(x, y) = Q(x + y) - Q(x) - Q(y))$, and for a lattice L in U dL stands for the discriminant of L. For two ordered sets $(a_1, a_2, \cdots, a_n)$, $(b_1, b_2, \cdots, b_n)$, we define the order $(a_1, a_2, \cdots, a_n) \leq (b_1, b_2, \cdots, b_n)$ by either $a_i = b_i$ for $i < k$ and $a_k < b_k$ for some $k \leq n$ or $a_i = b_i$ for any i.

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