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C^3-ACTIONS AND ALGEBRAIC THREEFOLDS WITH AMPLE TANGENT BUNDLE

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0. Introduction

One of the most challenging problems in complex differential geometry is the following conjecture of Frankel [3].

(F–n) *A compact Kaehler manifold M of dimension n with positive sectional (or more generally, positive holomorphic bisectional) curvature is biholomorphic to the complex projective space $\boldsymbol{P}^n(\boldsymbol{C})$.*

There are also algebraic counterparts:

(G–n) *A non-singular irreducible n-dimensional projective variety M with ample tangent bundle and the second Betti number* 1 *is isomorphic to $\boldsymbol{P}^n(\boldsymbol{C})$.*

(H–n) *A non-singular irreducible n-dimensional projective variety M with ample tangent bundle is isomorphic to $\boldsymbol{P}^n(\boldsymbol{C})$.*

The last (H–n) known as Hartshorne's Conjecture obviously implies (G–n). The first remarkable fact is that, for each n, Conjecture (G–n) implies (F–n); this is a consequence of the theorem of Bishop-Goldberg [1] and the celebrated theorem ("Every Hodge manifold is projective algebraic") of Kodaira [20]. (See also Goldberg-Kobayashi [9].)

We here give a historical sketch: (H–1) is straightforward from the fact that $\boldsymbol{P}^1(\boldsymbol{C})$ is the only compact Riemann surface with positive Euler number. Conjecture (F–2) was proved by Frankel and Andreotti [3], whereas Hartshorne [13] gave a purely algebraic proof of (H–2). Their proofs essentially depend on the classification of the rational algebraic surfaces.

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