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SCALAR EXTENSION OF QUADRATIC LATTICES

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Let E/F be a finite extension of algebraic number fields, O_E, O_F the maximal orders of E, F respectively. A classical theorem of Springer [6] asserts that an anisotropic quadratic space over F remains anisotropic over E if the degree $[E:F]$ is odd. From this follows that regular quadratic spaces U, V over F are isometric if they are isometric over E and $[E:F]$ is odd. Earnest and Hsia treated similar problems for the spinor genera [2, 3]. We are concerned with the quadratic lattices. Let L, M be quadratic lattices over O_F in regular quadratic spaces U, V over F respectively. Assume

(*) there is an isometry σ from O_EL onto O_EM, where O_EL, O_EM denote the tensor products of O_E and L, M over O_F respectively. Then our question is whether the assumption implies $\sigma(L) = M$ or not. The affirmative answer would imply that L, M are already isometric over O_F. Obviously the answer is negative if the quadratic space EU ($\cong EV$) is indefinite. Even if we suppose that EU is definite, the answer is still negative in general. However there are many cases in which the answer is affirmative if EU is definite. We give such examples in this paper.

Through this paper $Q(x), B(x, y)$ denote quadratic forms and corresponding bilinear forms ($2B(x, y) = Q(x + y) - Q(x) - Q(y)$). Notations and terminologies will be those of O'Meara [5].

THEOREM 1. *Let m be a natural number ≥ 2, and E be a totally real algebraic number field with degree m, and assume that L, M be definite quadratic lattices over the ring $\boldsymbol{Z}$ of rational integers. Then the assumption**) (*) *implies $\sigma(L) = M$ if E does not intersect with a finite set of (explicitely determined) algebraic integers which are not dependent on L, M, but on m.*

THEOREM 2. *Let E be totally real, and L, M be definite quadratic*

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*) In Theorem 1, 2, and 3 F is the field $\boldsymbol{Q}$ of rational numbers.