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Nagoya Math. J.
Vol. 64 (1976), 47-61

CHARACTER FORMULAS FOR DISCRETE SERIES ON SEMISIMPLE LIE GROUPS

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§ 1. Introduction

Let G be a connected, semisimple real Lie group with finite center, K a maximal compact subgroup of G. Assume $\operatorname{rank} G = \operatorname{rank} K$. Let $\mathfrak{G}$ be the Lie algebra of G, $\mathfrak{G}_C$ its complexification. If G_C is the simply connected complex analytic group corresponding to $\mathfrak{G}_C$, assume G is the real analytic subgroup of G_C corresponding to $\mathfrak{G}$.

In this case, G always has discrete series representations. The characters of these representations are distributions on the group G, realizable as locally integrable functions. Formulas for these characters are known up to certain integer constants which have only been evaluated for a few special cases. The purpose of this paper is to give information on how these constants can be computed in general, and to illustrate the method for several new cases.

For any Cartan subalgebra $\mathfrak{h}$ of $\mathfrak{G}$, let $\mathfrak{h}_C$ denote its complexification, $\Phi(\mathfrak{G}_C, \mathfrak{h}_C)$ the set of roots of the pair $(\mathfrak{G}_C, \mathfrak{h}_C)$, and $W(\mathfrak{G}_C, \mathfrak{h}_C)$ the Weyl group generated by the reflections corresponding to the roots. Let $\pi^{\mathfrak{h}}(H) = \prod \alpha(H)$, the product over all α in $\Phi^+(\mathfrak{G}_C, \mathfrak{h}_C)$, H any element of $\mathfrak{h}$.

Denote by $\mathfrak{k}$ the subalgebra of $\mathfrak{G}$ corresponding to K, and let $\mathfrak{t}$ be a Cartan subalgebra of $\mathfrak{G}$ such that $\mathfrak{t} \subseteq \mathfrak{k}$. Consider the space $\mathscr{F}$ of all pure imaginary linear functions on $\mathfrak{t}$. Let $\mathscr{F}' = \{\lambda \in \mathscr{F} : \langle \lambda, \alpha \rangle \neq 0$ for all $\alpha \in \Phi(\mathfrak{G}_C, \mathfrak{t}_C)\}$, the regular elements of $\mathscr{F}$. Then for each $\lambda \in \mathscr{F}'$ there exists a unique invariant distribution T_λ on $\mathfrak{G}$ characterized by certain properties [2a), p. 277].

Let W_K be the subgroup of $W(\mathfrak{G}_C, \mathfrak{t}_C)$ generated by reflections corresponding to the compact roots of $(\mathfrak{G}, \mathfrak{t})$. Then for $H \in \mathfrak{t}' = \mathfrak{t} \cap \mathfrak{G}'$, $\mathfrak{G}'$ the set of regular elements of $\mathfrak{G}$,

Received July 15, 1975.
* Supported in part by NSF grant MP572-05055 A02.