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# REPRESENTATIONS OF QUADRATIC FORMS AND THEIR APPLICATION TO SELBERG'S ZETA FUNCTIONS

***Dedicated to the memory of Taira Honda***

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Let $M$ and $L$ be quadratic lattices over the maximal order of an algebraic number field. In case of dealing with representations of $M$ by $L$, they sometimes assume certain indefiniteness and the condition rank $L$-rank $M \geq 3$. In this case, representation problems are reduced not to global but to local problems by virtue of the strong approximation theorem for rotations and of the fact that for regular quadratic spaces $U, V$ over a non-archimedian local field there is an isometry from $U$ to $V$ if $\dim V - \dim U \geq 3$. On the contrary, global properties seem to be strongly concerned if we omit one of those two assumptions. As an example we prove in § 1 that there is a sublattice of codimension 1 which characterizes $L$ in a certain sense. In § 2 we prove as its application that certain Selberg's zeta functions are linearly independent.

We denote by $\boldsymbol{Q}, \boldsymbol{Z}, \boldsymbol{Q}_p$ and $\boldsymbol{Z}_p$ the rational number field, the ring of rational integers, the $p$-adic completion of $\boldsymbol{Q}$, and the $p$-adic completion of $\boldsymbol{Z}$. We mean by a quadratic lattice $L$ over $\boldsymbol{Z}$ (resp. $\boldsymbol{Z}_p$) a $\boldsymbol{Z}$ (resp. $\boldsymbol{Z}_p$)-lattice in a regular quadratic space $U$ over $\boldsymbol{Q}$ (resp. $\boldsymbol{Q}_p$), and by definition $\operatorname{rank} L = \dim U$. For a quadratic lattice $L$ over $\boldsymbol{Z}$ (or $\boldsymbol{Z}_p$) we denote by $Q(x)$ and $B(x, y)$ the quadratic form and the bilinear form associated with $L$ ($2B(x, y) = Q(x + y) - Q(x) - Q(y)$), and by $dL$ the determinant of $(B(e_i, e_j))$ where $\{e_i\}$ is a basis of $L$ over $\boldsymbol{Z}$ (or $\boldsymbol{Z}_p$). $dL$ is uniquely determined for a quadratic lattice $L$ over $\boldsymbol{Z}$, and for a quadratic lattice $L$ over $\boldsymbol{Z}_p$, $dL$ is unique up to the squares of units in $\boldsymbol{Z}_p$. For two ordered sets $(a_1, a_2, \cdots, a_n)$, $(b_1, b_2, \cdots, b_n)$, we define the order $(a_1, a_2, \cdots, a_n) \leq (b_1, b_2, \cdots, b_n)$ by either $a_i = b_i$ for $i < k$ and $a_k < b_k$ for some $k \leq n$

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