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ON CLASSIFICATION OF QUASI-SYMMETRIC DOMAINS

Dedicated to the memory of Taira Honda

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The notion of "Siegel domains" was introduced by Pjateckiĭ-Šapiro [8]. It was then shown that every homogeneous bounded domain is holomorphically equivalent to a Siegel domain (of the second kind) determined uniquely up to an affine isomorphism ([15], cf. also [2], [4], [9b]). In a recent note [10b], I have shown that among (homogeneous) Siegel domains the symmetric domains can be characterized by three conditions (i), (ii), (iii) on the data (U, V, Ω, F) defining the Siegel domain (see Theorem in § 2 of this paper)[1]. The class of homogeneous Siegel domains satisfying partial conditions (i), (ii), which we propose to call "quasi-symmetric", seems to be of some interest, since for instance the fibers appearing in the expressions of symmetric domains as Siegel domains of the third kind fall in this class ([10b], [16]). Recently, using a method of S-algebras ([11a, b]), Takeuchi [11c] gave a complete classification of quasi-symmetric domains, which naturally implies a new classification of symmetric domains[2]. The purpose of the present note is to show that this classification can also be obtained immediately from my previous result on linear imbeddings of self-dual cones ([10a]).

Our method is based on the following two observations:

(I) There are natural equivalences between the three categories of (punctured) self-dual cones, the corresponding reductive Lie algebras (with fixed Cartan involutions), and formally real Jordan algebras (§ 1).

(II) There is a natural bijection between the set of isomorphism classes of quasi-symmetric Siegel domains and that of equivalence classes of the pairs formed of a self-dual cone and a (linear) "representation" of it (§ 3, Proposition 2).

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1) A similar result was also obtained independently by J. Dorfmeister.

2) Several results toward the classification of Siegel domains satisfying only the condition (i) have been obtained by Takeuchi [11b], Tsuji [13] and others.