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ON A HYPOELLIPTIC BOUNDARY VALUE PROBLEM

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§ 1. Introduction.

This paper is devoted to the investigation of the hypoellipticity of the following first boundary value problem:

$$
\begin{aligned}
(1.1) \quad Lu &= u_{tt} + (a(x,t)u_x)_x + g(x,t)u_{xt} + b(x,t)u_x + b^0(x,t)u_t + c(x,t)u \\
&= f(x,t) \qquad \text{in } \Omega ,
\end{aligned}
$$

$$
(1.2) \qquad u(x,t)|_{t=0} = 0 , \qquad |x| < R ,
$$

where Ω is an open rectangular domain in (x,t)-plane:

$$
\Omega = (-R < x < R) \times (0 < t < T) \qquad R > 0,\ T > 0 .
$$

We assume that the coefficients $a(x,t), b(x,t), b^0(x,t)$ and $c(x,t)$ are all C^∞ functions in $\bar{\Omega}$ satisfying the following conditions:

$$
(1.3) \qquad \operatorname{Re} a(x,t) \geqq 0 \qquad \text{in } \bar{\Omega} ,
$$

(1.4) for all x with $|x| < R$, the function $t \mapsto \operatorname{Re} a(x,t)$ has only finite zeros of order less than or equal to ℓ $(\geqq 0)$ in the interval $[0 \leqq t \leqq T]$

$$
(1.5) \qquad |\operatorname{Im} a(x,t)| \leqq C^{1)} \operatorname{Re} a(x,t) \qquad \text{in } \bar{\Omega}\ (C > 0) ,
$$

$$
(1.6) \qquad |\operatorname{Im} a_x(x,t)| \leqq C[\operatorname{Re} a(x,t)]^{1/2} \qquad \text{in } \bar{\Omega} ,
$$

$$
(1.7) \qquad t\,|\operatorname{Im} b(x,t)|^2 \leqq C \operatorname{Re} a(x,t) \qquad \text{in } \bar{\Omega} ,
$$

$$
(1.8) \qquad |g(x,t)| \leqq \frac{\varepsilon_1}{2}[\operatorname{Re} a(x,t)]^{1/2} \qquad \text{in } \bar{\Omega},\ 0 < \varepsilon_1 < 1 ,
$$

$$
(1.9) \qquad |g_t(x,t)| \leqq C[\operatorname{Re} a(x,t)]^{1/2} \qquad \text{in } \bar{\Omega} .
$$

We set $\tilde{\Omega} = (-R < x < R) \times [0 \leqq t < T)$. The main result of this paper is to prove the following theorem.

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1) We use the symbols $C, C^1, \ldots$ to express the different positive constants throughout this paper.