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STABLE VECTOR BUNDLES ON AN ALGEBRAIC SURFACE

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Introduction.

Let X be a non-singular projective algebraic curve over an algebraically closed field k. D. Mumford introduced the notion of stable vector bundles on X as follows;

DEFINITION ([7]). A vector bundle E on X is stable if and only if for any non-trivial quotient bundle F of E,

$$\deg(E)/r(E) < \deg(F)/r(F) ,$$

where $\deg(\cdot)$ denotes the degree of the first Chern class of a vector bundles and $r(\cdot)$ denotes the rank of a vector bundle.

D. Mumford, M. S. Narasimhan and C. S. Seshadri showed that the family of stable vector bundles on X with given degree and rank has a coarse moduli scheme ([7], [11], [12], [13]). To prove this they used some special facts which were provided by the assumption that X was a curve. For instance, (1) a coherent $\mathcal{O}_X$-module is torsion free if and only if it is locally free, (2) every vector bundle E has a filtration $0 = E_0 \subset E_1 \subset \cdots \subset E_{r-1} \subset E_r = E$ such that E_i/E_{i-1} is a locally free $\mathcal{O}_X$-module of rank 1, (3) the set of isomorphism classes of indecomposable vector bundles on X with fixed degree (Chern class) and rank is bounded[1].

Let us consider higher dimensional cases. Assume that X is a non-singular projective variety over k with $\dim X \geq 2$. Since, at least, the above three are not necessarily true, we have to overcome various difficulties to construct moduli of vector bundles on X. It is inevitable

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1) Let X be an algebraic scheme over an algebraically closed field k. We say a set S of coherent $\mathcal{O}_X$-modules is bounded if there exist an algebraic k-scheme T and a T-flat coherent $\mathcal{O}_{X \times_k T}$-module F such that every member of S is isomorphic to one of $\{F_t = F \otimes_{\mathcal{O}_{X \times T}} k(t) \mid t \in T(k)\}$.