L. A. Campbell and R. H. Ogawa
Nagoya Math. J.
Vol. 57 (1975), 37–47

ON PRESERVING THE KOBAYASHI PSEUDODISTANCE

L. ANDREW CAMPBELL AND ROY H. OGAWA

§ 0.

If X is a complex space, the Kobayashi pseudo-distance d_X is an intrinsic pseudometric on X defined as follows. If p and q are points of X, a chain α from p to q consists of intermediate points $p_0, \cdots, p_r$ with $p_0 = p$ and $p_r = q$ together with maps f_i of the unit disc $D = \{z \in C^1 \,|\, |z| < 1\}$ into X and points a_i and b_i in D such that $f_i(a_i) = p_{i-1}$ and $f_i(b_i) = p_i$ for $i = 1, \cdots, r$. If $d_D(a, b)$ denotes the hyperbolic distance between the points a and b in the unit disc, then the length of the chain α is defined as $|\alpha| = d_D(a_1, b_1) + d_D(a_2, b_2) + \cdots + d_D(a_r, b_r)$. The pseudo-distance between p and q is then defined as the infimum of the lengths of all chaini from p to q: $d_X = \inf\{|\alpha| \,|\, \alpha \text{ a chain from } p \text{ to } q\}$. It is easy to establish that $d_X(p, q)$ is jointly continuous in p and q and that holomorphic maps are distance decreasing—i.e. if $f: X' \to X$ is holomorphic and $f(p') = p$, $f(q') = q$ then $d_X(p, q) \leq d_{X'}(p', q')$. If d_X is an actual distance—i.e. if $d_X(p, q) \neq 0$ for $p \neq q$—then X is said to be hyperbolic and in that case the metric topology induced by d_X coincides with the original topology of X ([1]). A general reference for this subject is Kobayashi's book [4].

If A is a closed subset of X, then the inclusion map $X - A \to X$ is holomorphic, so that $d_X(p, q) \leq d_{X-A}(p, q)$ for p and q not in A. Removing an analytic set of codimension 1 often changes the pseudo-distance radically. For instance, the pseudo-distance on $C^* = \{z \in C \,|\, z \neq 0\}$ is identically zero, but, if we remove a single point from C^*, what is left is a hyperbolic space. The same sort of phenomenon generally does not occur if A is an analytic set of codimension at least 2. For instance, Kobayashi proves ([4]) that if A is closed and nowhere dense in some hyperplane section of D^n (the unit polydisc in n-space), then removing

Received June 5, 1974
Revised September 10, 1974