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ON LIE ALGEBRAS OF VECTOR FIELDS WITH INVARIANT SUBMANIFOLDS

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§ 0. Introduction.

It is known (Pursell and Shanks [9]) that an isomorphism between Lie algebras of infinitesimal automorphisms of C^∞ structures with compact support on manifolds M and M' yields an isomorphism between C^∞ structures of M and M'.

Omori [5] proved that this is still true for some other structures on manifolds. More precisely, let M and M' be Hausdorff and finite dimensional manifolds without boundary. Let α be one of the following structures:

(1) C^∞-structures, $(\alpha = \phi)$

(2) SL-structure, i.e. a volume element (positive n-form) with a non-zero constant multiplicative factor, $(\alpha = dV)$

(3) Sp-(symplectic) structure, i.e. symplectic 2-form with a non-zero constant multiplicative factor, $(\alpha = \Omega)$

(4) Contact structure, i.e. contact 1-form with a non-zero C^∞-function as a multiplicative factor, $(\alpha = \omega)$

(5) Fibring with compact fibre, $(\alpha = \mathscr{F})$

Let α (resp. α') be one of the above structures on M (resp. M'). Let $\Gamma_\alpha(T_M)$ be the Lie algebra of all C^∞, α-preserving infinitesimal transformations with compact support. We denote by $\mathscr{D}_\alpha(M)$ the group of all C^∞, α-preserving diffeomorphisms on M with compact support, that is, identity outside a compact subset. Then we have the following theorem

THEOREM (Omori [5]). *$\Gamma_\alpha(T_M)$ is algebraically isomorphic to $\Gamma_{\alpha'}(T_{M'})$, if and only if (M, α) is isomorphic to (M', α'). Especially, $\mathscr{D}_\alpha(M)$ is isomorphic to $\mathscr{D}_{\alpha'}(M')$.*

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