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A CLASS OF NONANALYTIC AUTOMORPHIC FUNCTIONS

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In this paper we consider a class of nonanalytic automorphic functions which were first mentioned to A. Selberg by C. L. Siegel. These functions have Fourier coefficients which are closely connected with the Fourier coefficients of analytic automorphic forms, and they are also eigenfunctions of the Laplace operator derived from the hyperbolic metric. We shall show how this latter property gives new results in the classical theory of automorphic forms.

I am indebted to Professor Selberg for introducing me to these functions.

§ 1. Definitions and Notation

Let Γ be a discrete subgroup of $SL(2, \boldsymbol{R})$ having a fundamental domain of finite non-Euclidean area. We assume that Γ has a single maximal parabolic subgroup $\Gamma_\infty = \left\{ \pm \begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix} : n \in \boldsymbol{Z} \right\}$, although this assumption is not necessary. Let $\mathscr{D}$ be the fundamental domain containing the strip $S_Y = \{z : \operatorname{Im} z > Y, 0 < \operatorname{Re} z < 1\}$, for sufficiently large positive Y.

If H denotes the upper half plane and z and z' are in H, where $z = x + iy$, $z' = x' + iy'$, then the non-Euclidean metric $d(z, z')$ is given by $\cosh d(z, z') = 1 + |z - z'|^2/2yy'$. The invariant measure is $dz = y^{-2}dxdy$ and the Laplace operator derived from the metric is

$$D = y^2\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) .$$

Let $L^2(\mathscr{D})$ be the space of all square-integrable automorphic functions f defined on H; i.e., $f(Mz) = f(z)$ for all $M \in \Gamma$ and $\int_{\mathscr{D}} |f(z)|^2 dz < \infty$. The inner product in this Hilbert space is denoted by $(f, g) = \int_{\mathscr{D}} f\bar{g} dz$, and the self-adjoint operator derived from D is also denoted by D.

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