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HIGHER RECIPROCITY LAW, MODULAR FORMS OF WEIGHT 1 AND ELLIPTIC CURVES

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§ 0. Introduction

In this paper, we study higher reciprocity law of irreducible polynomials $f(x)$ over $\boldsymbol{Q}$ of degree 3, especially, its close connection with elliptic curves rational over $\boldsymbol{Q}$ and cusp forms of weight 1. These topics were already studied separately in a special example by Chowla-Cowles [1] and Hiramatsu [2]. Here we bring these objects into unity.

Let

$\mathscr{C}_0 =$ the set of number fields K over $\boldsymbol{Q}$ such that
(1) K is a Galois extension over $\boldsymbol{Q}$ with $\mathrm{Gal}(K/\boldsymbol{Q}) \cong S_3$, the symmetric group of degree 3,
(2) K contains an imaginary quadratic field k.

For any K in $\mathscr{C}_0$, we can associate three other objects: (1) $f(x)$: irreducible polynomials over $\boldsymbol{Q}$ of degree 3, (2) $F(\tau)$: cusp forms of weight 1, (3) E: elliptic curves rational over $\boldsymbol{Q}$;
let

$\mathscr{C}_1 =$ the set of all irreducible polynomials $f(x)$ over $\boldsymbol{Q}$ of degree 3 whose splitting field K_f over $\boldsymbol{Q}$ belongs to $\mathscr{C}_0$.

$\mathscr{C}_2 =$ the set of all normalized cusp forms $F(\tau)$ of weight 1 on $\Gamma_0(N)$ whose Mellin transform is L-function with an ideal character χ of degree 3 of imaginary quadratic field k and the abelian extension K_F over k which corresponds to the kernel of χ belongs to $\mathscr{C}_0$.

$\mathscr{C}_3 =$ the set of all elliptic curves E rational over $\boldsymbol{Q}$ such that the field E_2 generated by coordinates of 2-division points on E belongs to $\mathscr{C}_0$.

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