T. Hida, K.-S. Lee and S.-S. Lee
Nagoya Math. J.
Vol. 98 (1985), 87-98

CONFORMAL INVARIANCE OF WHITE NOISE

TAKEYUKI HIDA, KE-SEUNG LEE AND SHEU-SAN LEE

§0. Introduction

The remarkable link between the structure of the white noise and that of the infinite dimensional rotation group has been exemplified by various approaches in probability theory and harmonic analysis. Such a link naturally becomes more intricate as the dimension of the time-parameter space of the white noise increases. One of the powerful method to illustrate this situation is to observe the structure of certain subgroups of the infinite dimensional rotation group that come from the diffeomorphisms of the time-parameter space, that is the time change. Indeed, those subgroups would shed light on the probabilistic meanings hidden behind the usual formal observations. Moreover, the subgroups often describe the way of dependency for Gaussian random fields formed from the white noise as the time-parameter runs over the basic parameter space.

The main purpose of this note is to introduce finite dimensional subgroups of the infinite dimensional rotation group that have important probabilistic meanings and to discuss their roles in probability theory. In particular, we shall see that the conformal invariance of white noise can be described in terms of the conformal group which is a finite dimensional Lie subgroup of the infinite dimensional rotation group.

As is well known, *the projective invariance* of the ordinary Brownian motion with one-dimensional parameter was discovered by P. Lévy [7], and a group theoretic as well as probabilistic interpretation was given in [6]. One may naturally ask "what is the higher dimensional parameter analogue of this property?" (See also [9]). This was the motivation of our present work. Our approach is rather group theoretic in technique, although it is probabilistic in spirit. For one thing, it is not so obvious to introduce a d-dimensional ($d \geq 2$) parameter analogue of a Brownian bridge, from which the discussion in [6] was originated. We shall there-

Received February 20, 1984.