H. Nakajima
Nagoya Math. J.
Vo. 98 (1985), 1-36

QUOTIENT COMPLETE INTERSECTIONS OF AFFINE SPACES BY FINITE LINEAR GROUPS

HARUHISA NAKAJIMA

§1. Introduction

Let G be a finite subgroup of $GL_n(C)$ acting naturally on an affine space C^n of dimension n over the complex number field C and denote by C^n/G the quotient variety of C^n under this action of G. The purpose of this paper is to determine G completely such that C^n/G is a complete intersection (abbrev. C.I.) i.e. its coordinate ring is a C.I. when $n > 10$. Our main result is (5.1). Since the subgroup N generated by all pseudo-reflections in G is a normal subgroup of G and C^n/G is obtained as the quotient variety of $C^n/N \cong C^n$ by G/N, without loss of generality, we may assume that G is a subgroup of $SL_n(C)$ (cf. [6, 16, 24, 25]).

Stanley classified G in [21] such that C^n/G is a C.I. under the assumption that $G = G^{\#} \cap SL_n(C)$ for a finite reflection group $G^{\#}$ in $GL_n(C)$, and conjectured in [23] that if C^n/G is a C.I., $G^* \supset G \supset [G^*, G^*]$ for a finite reflection group G^* in $GL_n(C)$. In [17, 28], this conjecture was solved negatively. On the other hand, Watanabe ([26]) and Watanabe-Rotillon ([29]) determined G such that C^n/G is a C.I. respectively for abelian G and for any G in $SL_3(C)$. In case of $n = 2$, it is well known and classical that C^2/G is always a hypersurface for every G in $SL_2(C)$.

Recently Goto and Watanabe showed that if C^n/G is a C.I., then its embedding dimension is at most $2n - 1$ i.e. C^n/G can be regarded as a closed subvariety of C^{2n-1} (cf. [27, 31]). This result follows from the main theorem in [11] on rational singularities, because C^n/G is a rational singularity at the induced origin (cf. [10]). Moreover, using Grothendieck's purity theorem, Kac and Watanabe [9] showed that if C^n/G is a C.I., then G is generated by $\{\sigma \in G \mid \dim \operatorname{Im}(\sigma - 1) \leqq 2\}$. Thanks to the last theorem, we can use a classification of some finite linear groups given by Blichfeldt, Huffman and Wales (see the references in [14]), and consequently, for example, have shown

Received August 8, 1983.