T. Higa
Nagoya Math. J.
Vol. 96 (1984), 41-60

ON THE TOPOLOGICAL STRUCTURE OF AFFINELY CONNECTED MANIFOLDS

TATSUO HIGA

Introduction

The purpose of the present paper is to investigate the relationship between the topological structure and differential geometric objects for affinely connected manifolds.

Let M be a compact, connected and oriented Riemannian manifold, $P^r(M)$ the vector space of all parallel r-forms on M and $b_r(M)$ the r-th Betti number of M. Since every parallel form is harmonic, it follows from the Hodge—de Rham theory that the inequality $\dim P^r(M) \leqq b_r(M)$ holds for all $r = 1, \cdots, \dim M$ (cf. [3], [5]). We shall generalize these inequalities to compact affinely connected manifolds.

Next, let M be a non-compact manifold. A connected submanifold N of M is called a *soul* if $\dim N < \dim M$ and if the inclusion $i: N \to M$ is a homotopy equivalence. J. Cheeger and D. Gromoll proved the following remarkable theorem. If M is a complete Riemannian manifold with non-negative sectional curvature then M has a compact soul (see [1] Theorem 1.11 and 2.1). We shall give another kind of sufficient conditions for M to have a (compact) soul.

Finally, a connected manifold M is said to be *reducible* if M is diffeomorphic to a product manifold $M_1 \times M_2$ with $\dim M_i \geqq 1$, $i = 1, 2$. Otherwise, M is said to be *irreducible*. We shall find a differential geometric condition for M to be reducible. We note that de Rham's Decomposition Theorem ([2]) furnishes a prototype for this condition (for irreducible manifolds, see [8]).

In order to obtain our results in a unified manner, we introduce certain family of functions on a connected manifold M with an affine connection Γ. A function f on M is called an *affine function* if, for every geodesic $c(t)$ with an affine parameter t, there are real constants a and b

Received July 8, 1983.