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ON A DUAL RELATION FOR ADDITION FORMULAS OF ADDITIVE GROUPS, I

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Introduction

This is the first in a series of papers concerned with a relation between a representation in the polynomial ring of additive groups and its translation invariant operators. The present study is to observe several properties of a polynomial sequence $p_\alpha(x)$ satisfying the binomial identity:

$$(1) \qquad p_\alpha(x+y) = \sum_{\alpha=\beta+\gamma} p_\beta(x) p_\gamma(y) ,$$

by means of some translation invariant operators. For example, to take the very simple case, that is, $x^\alpha/\alpha!$, the set of translation invariant operators is $\{\partial/\partial x_1, \cdots, \partial/\partial x_n\}$. This technique is the so called "umbral calculus" or "symbol calculus" widely used in the past century (cf. [2], [7]). This gives an effective technique for expressing a set of polynomials in terms of another.

In this series, we call it the dual relation for addition formulas of the additive groups. In the case of a polynomial sequence of one variable, G.C. Rota etc. [9] deals with the dual relation. In this series, we investigate the case of generic n variables.

Let us give a brief description of the contents of this paper.

Section 1 deals with one to one correspondence between a polynomial sequence with the binomial identity (1) and a set of n translation invariant operators.

Section 2 deals with the expansion theorem by a polynomial sequence with the binomial identity (1). Then as corollaries of this theorem, we obtain a characterization of the polynomial sequence by a numerical sequence, and a generating function of the polynomial sequence.

Section 3, that is a main result, deals with an analogy of the classical

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