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Nagoya Math. J.
Vol. 94 (1984), 75-87

A CONNECTION BETWEEN BLOWING-UP AND GLUINGS IN ONE-DIMENSIONAL RINGS

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Introduction

Let C be an affine curve, contained on a non-singular surface X as a closed 1-dimensional subscheme. If P is a closed point on C, the blowing-up C' of C with center P (induced by the blowing-up of X with center P) is an affine curve. It is known that there is a sequence:

$$(\cdot) \qquad \bar{C} = C_k \longrightarrow C_{k-1} \longrightarrow \cdots \longrightarrow C_1 \longrightarrow C_0 = C\,,$$

where $\bar{C}$ is the normalization of C, and each C_{i+1} is the blowing-up of C_i with center a singular point P_i on C_i ($i = 0, \cdots, k-1$).

The sequence $(\cdot)$ induces a sequence of rings:

$$(*) \qquad R = R_0 \subset R_1 \subset \cdots \subset R_{k-1} \subset R_k = \bar{R}\,,$$

where, for $j = 0, \cdots, k$, R_j is the coordinate ring of C_j; for each $i = 0, \cdots, k-1$, R_{i+1} is called the ring "obtained from R_i by blowing-up the maximal ideal of R_i corresponding to P_i".

On the other hand, there is also a sequence between R and $\bar{R}$:

$$(**) \qquad R = B_n \subset B_{n-1} \subset \cdots \subset B_1 \subset B_0 = \bar{R}\,,$$

where each B_{i+1} (i $= 0, \cdots, n-1$) is a "gluing of primary ideals of B_i over a prime ideal of R" (see [6]).

In this paper we wonder under what assumptions a sequence $(*)$ is also a sequence $(**)$ of gluings between R and $\bar{R}$; in this case, the method of "gluing" defined in [6] is "inverse" of the process of "blowing-up" used to obtain the desingularization of C. We give necessary and sufficient conditions on $(*)$ in order that $(*)$ is also a sequence of gluings like $(**)$; then, we show some classes of rings satisfying the required condition, in particular the rings considered in the last theorem of [7].

Received October 5, 1982.
This work was supported by C.N.R. (Consiglio Nazionale delle Ricerche).