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THE INVARIANT POLYNOMIAL ALGEBRAS FOR THE GROUPS $IU(n)$ AND $ISO(n)$

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§1. Introduction

By the coadjoint representation of a connected Lie group G with the Lie algebra $\mathfrak{g}$ we mean the representation $\mathrm{CoAd}(g) = {}^t\mathrm{Ad}\,(g^{-1})$ in the dual space $\mathfrak{g}^*$. Imitating Chevalley's argument for complex semi-simple Lie algebras, we shall show that the $\mathrm{CoAd}\,(G)$-invariant polynomial algebra on $\mathfrak{g}^*$ is finitely generated by algebraically independent polynomials when G is the inhomogeneous linear group $IU(n)$ or $ISO(n)$. In view of a well-known theorem [8, p. 183] our results imply that the centers of the enveloping algebras for the (or the complexified) Lie algebras of these groups are also finitely generated. Recently much more inhomogeneous groups have been studied in a similar context [2]. Our results, however, are further reaching as far as the groups $IU(n)$ and $ISO(n)$ are concerned [cf. 3, 4, 6, 7, 9].

We shall state our results.

(i) $IU(n)$ $(n\geqq 2)$.

Let G_n and $\mathfrak{g}_n$ be the group $IU(n)$ and its Lie algebra respectively, namely

$$G_n = \left\{\begin{pmatrix} u & a \\ 0 & 1\end{pmatrix};\ u \in U(n),\ a \in C^n\right\},$$

$$\mathfrak{g}_n = \left\{\begin{pmatrix} X & x \\ 0 & 0\end{pmatrix};\ X \in u(n),\ x \in C^n\right\}.$$

The dual space $\mathfrak{g}_n^*$ of $\mathfrak{g}_n$ can be identified with $\mathfrak{g}_n$ by the following non-degenerate bilinear form $\langle\ ,\ \rangle_n$ on $\mathfrak{g}_n\times\mathfrak{g}_n$;

$$\left\langle\begin{pmatrix} X & x \\ 0 & 0\end{pmatrix}, \begin{pmatrix} Y & y \\ 0 & 0\end{pmatrix}\right\rangle_n = \left\langle\begin{pmatrix} X & 0 \\ 0 & -\mathrm{tr}\,X\end{pmatrix}, \begin{pmatrix} Y & 0 \\ 0 & -\mathrm{tr}\,Y\end{pmatrix}\right\rangle_{su(n+1)} + \langle x, y\rangle\,,$$

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