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Nagoya Math J.
Vol. 93 (1984), 17-26

A MORDELL-WEIL GROUP OF RANK 8, AND A SUBGROUP OF FINITE INDEX

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It is well known [c.f. Kas] that every elliptic surface, with geometric genus 0, is given by a Weierstrass equation of the form

$$y^2 = 4x^3 - \sum_{i=0}^{4} a_i u^i x - \sum_{j=0}^{6} b_j u^j \tag{1}$$

(relative to a suitable parameter, u, for the base) where the a's and b's are constants. For sufficiently general choices of a's and b's, the Mordell-Weil group (i.e., the group of solutions (x, y), with x and y rational functions of u) has rank 8.

We will find, for a specific equation of this form,

$$y^2 = 4(x^3 - u^4 x + 1)\,, \tag{2}$$

8 solutions that generate a subgroup of index 4 in the Mordell-Weil group of the fibration given by this equation. We do this using the Cox-Zucker Machine. We then use this result to draw certain conclusions concerning the general case, and to make certain conjectures.

§1. The algorithm of Cox and Zucker (AKA, The Cox-Zucker Machine)

The purpose of this section is to provide, not a complete description of the algorithm of Cox and Zucker, but rather, a brief summary of their technique. See [1] for details.

Let $E \xrightarrow{f} B$ be an elliptic fibration with geometric genus zero; let $\sigma_0\colon B \to E$ be the zero section, i.e., the identity element of the Mordell-Weil group; let (,) be the intersection product on the space of 1-cycles on the surface, E; and let $S \subset B$ be the finite set of points of B, above which lie the singular fibers.

Received June 21, 1982.