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NON-LINEAR PREDICTION PROBLEMS FOR ORNSTEIN-UHLENBECK PROCESS

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§ 0. Introduction

We shall discuss in this paper some problems in non-linear prediction theory. An Ornstein-Uhlenbeck process $\{U(t)\}$ is taken to be a basic process, and we shall deal with stochastic processes $X(t)$ that are transformed by functions f satisfying certain condition. Actually, observed processes are expressed in the form $X(t) = f(U(t))$. Our main problem is to obtain the best non-linear predictor $\hat{X}(t, \tau)$ for $X(t + \tau)$, $\tau > 0$, assuming that $X(s)$, $s \leq t$, are observed. The predictor is therefore a non-linear functional of the values $X(s)$, $s \leq t$.

Non-linear prediction theory that discusses how to obtain such non-linear predictors has been considered in various situations. For instance, I. I. Gihman and A. V. Skorohod (cf. [1], § 8, Chapter IV, Vol. I) have considered optimum mean square predictor of $X(t + \tau)$, $\tau > 0$, assuming that the basic process $V(s)$, $s \leq t$, itself is observed. As is well-known the predictor $\hat{X}(t, \tau)$ is given by the conditional expectation:

$$(1) \qquad \hat{X}(t, \tau) = E\{X(t + \tau) \,|\, \mathscr{B}_t(V)\}\,, \qquad \mathscr{B}_t(V) = \sigma\{V(s);\; s \leq t\}\,.$$

While A. M. Yaglom [5] has discussed the optimum mean square predictor assuming that Markov process $V(t)$ is transformed by a function f with inverse f^{-1} and that $X(s) = f(V(s))$, $s \leq t$, are observed.

In this case, it holds evidently that

$$(2) \qquad \begin{aligned} \hat{X}(t, \tau) &= E\{X(t + \tau) \,|\, X(s);\; s \leq t\} = E\{X(t + \tau) \,|\, X(t)\} \\ &= E\{X(t + \tau) \,|\, V(t)\} = E\{X(t + \tau) \,|\, V(s);\; s \leq t\}\,. \end{aligned}$$

Yaglom's situation coincides with ours in the sense that the $X(s)$ are assumed to be given for $s \leq t$. In this case, too, the predictor (2) coincides with (1) actually, because f is invertible.

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