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GALOIS ACTION ON SOME IDEAL SECTION POINTS OF THE ABELIAN VARIETY ASSOCIATED WITH A MODULAR FORM AND ITS APPLICATION

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Introduction

For an integer N, let $X_1(N)$ be the modular curve defined over $\boldsymbol{Q}$ which corresponds to the modular group $\Gamma_1(N)$. To each primitive cusp form $f = \sum a_m q^m$, $a_1 = 1$, (= normalized new form in the sense of [1]) on $\Gamma_1(N)$ of weight 2, there corresponds a factor J_f of the jacobian variety of $X_1(N)$ (cf. Shimura [19]). Shimura [20] and Ohta [11] etc. investigated the Galois action on some ideal section points of J_f. They treated the case when f is a primitive cusp form on $\Gamma_1(l)$ with the neben typus character $\left(\frac{l}{-}\right)$ for a prime number l, $l \equiv 1 \bmod 4$. We here treat the forms on $\Gamma_0(l^n)$ (*i.e.*, the Haupt form) for a prime number $l \neq 2$. Put $K_f = \boldsymbol{Q}(a_m \mid 1 \leqq m \in \boldsymbol{Z})$ and δ_f be the ideal of the ring of integers $\mathcal{O}$ of K_f generated by a_q for all primes q such that $\left(\frac{\pm l}{q}\right) = -1$. Here, the sign $\pm$ is chosen so that $\pm l \equiv 1 \bmod 4$. When a form f is associated with a Grössen-character of an imaginary quadratic field (cf. [18]), we say that f has C.M. or f is a form with C.M. One of the results is the following, which was conjectured in Saito [17]:

PROPOSITION (cf. (1.10), (1.16)). *Let f be a primitive cusp form on $\Gamma_0(l^n)$ of weight 2 for a prime number l, $l \equiv -1 \bmod 4$. Assume tket there exists a prime $\mathfrak{P}$ of K_f which divides δ_f but not divide $2l$. Then, there exists a primitive cusp form Θ with C.M. on $\Gamma_0(l^n)$ of weight 2 such that*

$$f \equiv \Theta \bmod \overline{\mathfrak{P}},$$

where $\overline{\mathfrak{P}}$ is an extension of $\mathfrak{P}$ to $\overline{\boldsymbol{Q}}$. Further, if $\mathfrak{P} \nmid (l-1)\cdot l$, f and Θ belong to the same direct factor in Saito's decomposition of the space $S_2^0(\Gamma_0(l^n))$

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