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LÉVY'S FUNCTIONAL ANALYSIS IN TERMS OF AN INFINITE DIMENSIONAL BROWNIAN MOTION III

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§ 0. Introduction

The purpose of this paper is to define minimality of surfaces in an infinite dimensional space $\boldsymbol{E}$ by probabilistic methods with the description of the relation between minimal surfaces and harmonic functions on the space $\boldsymbol{E}$, and to analyze purely analytic properties of a certain class of quadratic forms on the space $\boldsymbol{E}$.

We have constructed in the previous papers (Hasegawa [3], [4])[1] an infinite dimensional sequence space $\boldsymbol{E}$:

$$(0.1) \qquad \boldsymbol{E} = \{x = (x_1, \cdots, x_n, \cdots) \in R^\infty; \sup_N (x_1^2 + \cdots + x_N^2)/N < \infty\},$$

a system of semi-norms $\{\| \ \|_N;\ 1 \leqslant N \leqslant \infty\}$:

$$(0.2) \qquad \|x\|_N = [(x_1^2 + \cdots + x_N^2)/N]^{1/2} \quad \text{and} \quad \|x\|_\infty = \overline{\lim_{N\uparrow\infty}} \|x\|_N ,$$

and an infinite dimensional Brownian motion $\boldsymbol{B} = (\Omega, B(t, \omega), P^x)$ on the space $\boldsymbol{E}$:

$$(0.3) \qquad B(t, \omega) = (b_1(t, \omega), \cdots, b_n(t, \omega), \cdots) \in \boldsymbol{E},$$

where $\{b_n(t, \omega); n \geqslant 1\}$ is a family of mutually independent 1-dimensional Brownian motions. The Laplacian $\triangle_\infty$ on the space $\boldsymbol{E}$ is defined as the infinitesimal generator of the Brownian motion $\boldsymbol{B}$ up to constant 1/2. Then we have interpreted some peculiar phenomena to P.Lévy's potential theory on the real Hilbert space $L^2([0, 1])$ mainly through the semi-norm $\| \ \|_\infty$. In this paper we shall describe other peculiarities to his theory again with the aid of the semi-norm $\| \ \|_\infty$.

Lévy has introduced to his theory important concepts in the geometry of the space $L^2([0, 1])$, i.e., the curvatures, in particular the mean curvature

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[1] Without special mentions we shall use the terminologies in Hasegawa [3], [4].