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RIEMANNIAN FOLIATIONS WITH PARALLEL CURVATURE

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§ 1. Introduction

Let M be a smooth compact manifold and let $\mathscr{F}$ be a smooth codimension q Riemannian foliation of M. Let $T(M)$ be the tangent bundle of M and let $E \subset T(M)$ be the subbundle tangent to $\mathscr{F}$. We may regard the normal bundle $Q = T(M)/E$ of $\mathscr{F}$ as a subbundle of $T(M)$ satisfying $T(M) = E \oplus Q$. Let g be a smooth Riemannian metric on Q invariant under the natural parallelism along the leaves of $\mathscr{F}$. This is equivalent to the existence of a bundle-like metric [16] and to the existence of a transverse $O(q)$-structure [5]. Recall that a connection ∇ on Q is basic if the induced parallel translation along a path lying in a leaf of $\mathscr{F}$ agrees with the natural parallelism along the leaves and that such a connection is characterized by the condition that $\nabla_X Y = [X, Y]_Q$ for all vector fields X tangent to E and Y tangent to Q where $[X, Y]_Q$ denotes the Q-component of the Lie bracket of X and Y [3]. The torsion of ∇ is the tensor field of type (1, 2) on M defined by $T(X, Y) = \nabla_X Y_Q - \nabla_Y X_Q - [X, Y]_Q$ where X and Y are vector fields on M. There is a unique torsion-free metric-preserving basic connection ∇ on Q [9], [11] defined as follows. Let $x \in M$. Let $f: U \to V$ be a submersion whose level sets are the leaves of $\mathscr{F}|U$ where U is a neighborhood of x in M and V is an open set in R^q. There is a unique Riemannian metric $\bar{g}$ on V such that $f^*(\bar{g}) = g|U$. Let $\bar{\nabla}$ be the Riemannian connection on V. Then $\nabla|U = f^{-1}(\bar{\nabla})$. It is natural to study the relationship between the curvature of ∇ and the structure of the foliated manifold $(M, \mathscr{F})$.

In the present work we study the case of parallel curvature, that is $\nabla R = 0$ where $R(X, Y)Z$ denotes the curvature tensor of ∇.

Let $\mathscr{F}$ be a Riemannian foliation with parallel curvature of a compact manifold M.

THEOREM 1. *Let $\tilde{M}$ be the universal cover of M and let $\tilde{\mathscr{F}}$ be the lift*

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