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CENTRAL EXTENSIONS AND SCHUR'S MULTIPLICATORS OF GALOIS GROUPS

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Introduction

When he developed the theory of central extensions of absolute abelian fields in [1], Fröhlich clearly pointed out a role of Schur's multiplicators of the Galois groups in algebraic number theory. Another role of them was to be well known when the gaps between the everywhere local norms and the global norms of finite Galois extensions were cohomologically described by Tate [10]. The relation of two roles was investigated by Furuta [2], Shirai [9], Heider [3] and others.

Let K/k be a finite Galois extension of algebraic number fields with $\mathfrak{g} = \mathrm{Gal}(K/k)$. Then Schur's multiplicator of $\mathfrak{g}$ is $\mathrm{H}^2(\mathfrak{g}, \boldsymbol{C}^\times)$ under the trivial action of $\mathfrak{g}$ on $\boldsymbol{C}^\times$. We may replace $\boldsymbol{C}^\times$ by the subgroup consisting of all the roots of 1 in $\boldsymbol{C}^\times$, which is naturally isomorphic to the additive group $\boldsymbol{Q}/\boldsymbol{Z}$. Then the derived exact sequence from $0 \to \boldsymbol{Z} \to \boldsymbol{Q} \to \boldsymbol{Q}/\boldsymbol{Z} \to 0$ gives an isomorphism of $\mathrm{H}^2(\mathfrak{g}, \boldsymbol{C}^\times) \simeq \mathrm{H}^2(\mathfrak{g}, \boldsymbol{Q}/\boldsymbol{Z})$ onto $\mathrm{H}^3(\mathfrak{g}, \boldsymbol{Z})$, which is the dual of $\mathrm{H}^{-3}(\mathfrak{g}, \boldsymbol{Z})$. The works of Furuta, Shirai and Heider were based on the fact that $\mathrm{H}^{-3}(\mathfrak{g}, \boldsymbol{Z})$ is isomorphic to $\mathrm{H}^{-1}(\mathfrak{g}, K_A^\times/K^\times)$ where $K_A^\times/K^\times$ is the idele class group of K.

Let $\bar{k}$ be the algebraic closure of k, and k_{ab} the maximal abelian extension of k in $\bar{k}$. Heider [3] showed that there always exists a finite central extension L of K/k such that $\mathrm{Gal}(L/L \cap (k_{\mathrm{ab}} \cdot K))$ is isomorphic to the dual of $\mathrm{H}^2(\mathfrak{g}, \boldsymbol{Q}/\boldsymbol{Z})$, and, by this fact, gave a new proof Tate's theorem which claims that $\mathrm{H}^2(\mathrm{Gal}(\bar{k}/k), \boldsymbol{Q}/\boldsymbol{Z}) = 0$. (One can see a proof of this theorem in Serre [8].)

In this paper, we start with Tate's theorem to clarify the above mentioned roles of Schur's multiplicators of Galois groups using Hochschild-Serre exact sequences of a simple case, and, as a consequence, give a simple proof of Heider's theorem.

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