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ON EXISTENCE OF TOLERANCE STABLE DIFFEOMORPHISMS*

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§ 1. Introduction

We consider a compact smooth manifold M. $\mathrm{Diff}^1(M)$ denotes the space of C^1-diffeomorphisms of M onto itself with the usual C^1-topology. In the research of the qualitative theory of dynamical systems there is a desire to find a concept of stability of geometric global structure of orbits such that this stable systems are dense in the space of dynamical systems on M. Structural stability does not satisfy the density condition in $\mathrm{Diff}^1(M)$. Tolerance stability (see Section 2 for definition) is a candidate for the density property [7, p. 294]. Concerning tolerance stability there are researches as [6], [7], [8], and [2].

If $f \in \mathrm{Diff}^1(M)$ is structurally stable in strong sense, f is topologically stable in $\mathrm{Diff}^1(M)$ (see Section 2 for definition). Moreover, topological stability implies tolerance stability [A. Morimoto, 2]. The proof of this property will be introduced in Section 2.

The main result of this paper is to show the existence of diffeomorphisms on any compact manifold M which are tolerance stable but not topologically stable in $\mathrm{Diff}^1(M)$, so that, not structurally stable in strong sense. This will be proved in Sections 3, 4 and 5.

§ 2. Definitions and statement of results

We denote by $\mathrm{Homeo}(M)$ the set of homeomorphisms of M onto itself; the topology on $\mathrm{Homeo}(M)$ is given by the neighborhood $N_\varepsilon(f)$ of $f \in \mathrm{Homeo}(M)$

$$N_\varepsilon(f) = \{g;\, d(f, g) < \varepsilon\}, \qquad \varepsilon > 0\,.$$

Here, for a Riemannian metric d on M, $d(f, g) < \varepsilon$ means

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