A. Sannami
Nagoya Math. J.
Vol. 90 (1983), 1–55

THE STABILITY THEOREMS FOR DISCRETE DYNAMICAL SYSTEMS ON TWO-DIMENSIONAL MANIFOLDS

ATSURO SANNAMI

§1. Introduction

One of the basic problems in the theory of dynamical systems is the characterization of stable systems.

Let M be a closed (i.e. compact without boundary) connected smooth manifold with a smooth Riemannian metric and $\mathrm{Diff}^r(M)$ $(r \geq 1)$ denote the space of C^r diffeomorphisms on M with the uniform C^r topology. Let $f \in \mathrm{Diff}^s(M)$ with $s \geq r$. Then f is called C^r structurally stable if and only if there is a neighborhood $\mathscr{U}(f)$ of f in $\mathrm{Diff}^r(M)$ such that for any $g \in \mathscr{U}(f)$ there exists a homeomorphism $h: M \to M$ satisfying $gh = hf$.

Another important notion of stability is the Ω-stability. Recall that $x \in M$ is a non-wandering point of f if and only if for any neighborhood U of x, there is a nonzero integer m such that $f^m(U) \cap U \neq \phi$. The set $\Omega(f)$ of all the non-wandering points of f is a closed invariant set. f is called C^r Ω-stable if and only if there is a neighborhood $\mathscr{U}(f)$ of f in $\mathrm{Diff}^r(M)$ such that for any $g \in \mathscr{U}(f)$ there exists a homeomorphism $h: \Omega(f) \to \Omega(g)$ satisfying $gh = hf$ on $\Omega(f)$.

The essential condition to characterize these stabilities is "Axiom A" introduced by S. Smale in [17]. Namely, f satisfies Axiom A if and only if

(a) $\Omega(f)$ is a hyperbolic set,

(b) $\overline{\mathrm{Per}(f)} = \Omega(f)$,

where $\mathrm{Per}(f)$ denotes the set of all the periodic points of f. Recall that a compact f-invariant subset $\Lambda \subset M$ is a hyperbolic set if and only if there exist constants $c > 0$, $0 < \lambda < 1$ and a Tf-invariant splitting $TM|\Lambda = E^s \oplus E^u$ such that

$$\|Tf^n | E^s_p\| \leq c\lambda^n$$
$$\|Tf^{-n} | E^u_p\| \leq c\lambda^n$$

for all $p \in \Lambda$ and non-negative integers n.

Received April 22, 1981.