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# EXTERIOR POWERS OF FIELDS AND SUBFIELDS

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***Dedicated to Professor Goro Azumaya on the occasion of his 60th birthday***

## Introduction

Let $D$ be a division algebra of finite dimension $n^2$ over it's center $F$. Suppose $D$ has an involution, $\tau$, of the first kind, of symplectic type (e.g. [1], p. 169). By the theory of the pfaffian, $\tau$ symmetric elements have degree less than $n/2$ over $F$. On the other hand, Tamagawa has shown (unpublished) that involutions like $\tau$ are closely related to minimal symmetric idempotents in $D \otimes_F D$. This author began by examining and trying to generalize these relationships. But before any theory seemed possible for division algebras, a theory relating subfields and symmetric idempotents was required. This investigation gave rise to the results presented here, especially the main theorem in Section Two.

We begin by considering a finite separable field extension $L/F$ and it's tensor power $T_m(L/F) = L \otimes_F \cdots \otimes_F L$ ($m$ times). Already from Tamagawa's work, it is clear that not all minimal symmetric idempotents are of interest to us, but only those corresponding to symplectic involutions. It turns out that what we are actually interested in is an $F$ algebra $E_m(L/F)$ closely related to the exterior power $\Lambda^m L$. This is the exterior power of the title. In Section Two we prove a correspondence theorem relating idempotents of $E_m(L/F)$ and subfields of $L$. Specifically, there is a one to one correspondence between subfields $L' \subseteq L$ of codimension $m$ and certain minimal-like idempotents of $E_m(L/F)$. This correspondence theorem generalizes the facts concerning the pfaffian mentioned above.

Assuming $L/F$ is a separable field extension is unnaturally restrictive. It better serves our purpose to assume that $L$ is a separable commutative algebra over $F$, and that $F$ is a finite direct sum of fields. We do so assume in all of this paper. An $F$ module $V$ may not be a free $F$ module,

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