H. Kaneta
Nagoya Math. J.
Vol. 87 (1982), 147-174

IRREDUCIBILITY OF SOME UNITARY REPRESENTATIONS OF THE POINCARÉ GROUP WITH RESPECT TO THE POINCARÉ SUBSEMIGROUP, II

HITOSHI KANETA

Let $P(3)$ and $P_+(3)$ be the 3-dimensional space-time Poincaré group and the Poincaré subsemigroup, that is, $P(3) = R^3 \times_s SU(1,1)$ and $P_+(3) = V_+(3) \times_s SU(1,1)$ where $V_+(3) = \{x_0^2 - x_1^2 - x_2^2 \geq 0, x_0 \geq 0\}$. The multiplication is defined by the formula $(x, g)(x', g') = (x + g^{*-1}x'g^{-1}, gg')$ for $x, x' \in R^3$ and $g, g' \in SU(1,1)$. Here $x = (x_0, x_1, x_2)$ is identified with the matrix $\begin{pmatrix} x_0 & x_2 - ix_1 \\ x_2 + ix_1 & x_0 \end{pmatrix}$.

The purpose of this paper is to give an affirmative answer to the problem if there is any irreducible unitary representation of $P(3)$ such that its restriction to the semigroup $P_+(3)$ is reducible. To be more precise, we shall determine all $P_+(3)$-invariant, closed proper subspaces for the irreducible unitary representations $(U^{\eta,\varepsilon}, \mathfrak{H}^{\eta,\varepsilon})$ $(\eta \in R, \varepsilon = 0, 1/2)$, which are associated with the one-sheeted hyperboloid $V_{iM}(3) = \{y_0^2 - y_1^2 - y_2^2 = -M^2\}$ $(M > 0)$. As for the other irreducible unitary representations of $P(3)$ it is easy to show that they are irreducible even when they are restricted to $P_+(3)$ (see [5], Theorem 5). Recall that all the irreducible unitary representations of the 2-dimensional space-time Poincaré group are irreducible even when they are restricted to the Poincaré subsemigroup ([5], Theorem 1). Using, among other things, the results in § 1, we shall show in the forthcoming Part III that the irreducible unitary representations of the 4-dimensional space-time Poincaré group whose irreducibility relative to the Poincaré subsemigroup remains unsettled in [5] are reducible as the representations of the semigroup.

The basic tools of our approach are i) the eigenfunction expansions for second order ordinary differential operators $\mathscr{L}_{k,\eta}$ (see (1.1)), which are connected with the Laplacian of $SU(1,1)$, and ii) rephrased versions of the Hilbert transform and the Frobenius method for ordinary differential

Received November 26, 1980.