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DEFINABILITY THEOREM FOR THE INTUITIONISTIC PREDICATE LOGIC WITH EQUALITY

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Introduction

Svenonius' definability theorem and its generalizations to the infinitary logic $L_{\omega_1\omega}$ or to a second order logic with countable conjunctions and disjunctions have been studied by Kochen [1], Motohashi [2], [3] or Harnik and Makkai [4], independently. In this paper, we consider a (Svenonius-type) definability theorem for the intuitionistic predicate logic IL with equality.

First we recall Svenonius' theorem and Motohashi's theorem. Suppose that L_1 is a first order logic with equality, L_2 is a second order logic with countable conjunctions and disjunctions and L is either L_1 or L_2. Let P be a k-ary predicate constant not in L, $T(P)$ a set of sentences (resp. negative sentences) in $L_1(P)$ (resp. $L_2(P)$). In the case of $L = L_2$, we assume that the set of individual free variables are divided into two infinite disjoint sets X and Y. Now, consider the following three conditions:

(i) *For any models* $\mathfrak{a}$, $\mathfrak{b}$ *of* $T(P)$, $\mathfrak{a}|L = \mathfrak{b}|L$ *and* $\mathfrak{a} \cong \mathfrak{b}$ *imply* $\mathfrak{a} = \mathfrak{b}$.

(ii) $T(P) \vdash_{L(P)} \bigvee_{i=1}^{n} (\forall \bar{u})(P(\bar{u}) \equiv \varphi_i(\bar{u}))$ *for some formulas* $\varphi_1(\bar{x}), \cdots, \varphi_n(\bar{x})$ *in* L.

(iii) $T(P) \vdash_{L(P)} (\forall \bar{u})(P(\bar{u}) \equiv \varphi(\bar{u}))$ *for some Motohashi P-formula* $\varphi(\bar{x})$ *in* $L(P)$ *whose free variables are among* $\bar{x} \subseteq X$.

(See [3] or [4] about Motohashi P-formula in $L(P)$.)
Then, Svenonius' theorem is that the conditions (i) and (ii) are equivalent in the case of $L = L_1$ and Motohashi's theorem is that the conditions (i) and (iii), hence also (ii), are all equivalent in the case of $L = L_2$.

When we study the relations between these conditions for the intuitionistic predicate logic IL, we must consider the following syntactical condition (i)$'$ instead of the semantical condition (i).

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