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EXISTENCE AND BIFURCATION OF SOLUTIONS FOR FREDHOLM OPERATORS WITH NONLINEAR PERTURBATIONS

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Introduction

In this paper we shall discuss nonlinear eigenvalue problems for the equations of the form

$$(1) \qquad Lx + \lambda K(x) - M(x, \lambda) = 0\,, \quad x \in X\,, \quad \lambda \in R\,,$$

where L is a linear operator on a real Banach space X with non-zero kernel, $K(\cdot)$ is a linear or nonlinear operator on X and $M(\cdot, \cdot)$ is an operator from $X \times R$ into X. Equations of the form (1) arise in various fields of physics and engineering. For example, if $L = \Delta - \mu$, $K(x) = f|x|^{k-1}x$ and $M(x, \lambda) = g|x|^{m-1}x$, then the equation (1) is the nonlinear stationary equation of the Klein-Gordon type.

A solution of (1) means a pair $(x, \lambda) \in X \times R$ satisfying the equation (1). The main purpose of this paper is to prove the existence of solutions of (1) and to investigate the local structure of the solution sets.

An important case is the one where $K(0) = 0$ and $M(x, \lambda) = o(\|x\|)$ uniformly in $\lambda \in \Lambda$, Λ being an interval containing zero. Clearly, $(0, \lambda)$, for any $\lambda \in \Lambda$, is a solution of (1); this solution is called a trivial solution. We are interested in determining conditions for the existence of nontrivial solutions of (1).

We say that $(0, 0)$ is a bifurcation point of (1) with respect to the line of trivial solutions, if every neighbourhood of $(0, 0)$ in $X \times R$ contains non-trivial solutions. The bifurcation problems which are reduced to equations of the type (1) have been discussed by many authors. For example, Rabinowitz [7] has considered the case where $L = I + K$ with K being compact and linear. Ize [2] has also treated the case where L is a Fredholm operator of index zero and K is the identity operator. They

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