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ON THE DISTRIBUTION (MOD 1) OF POLYNOMIALS OF A PRIME VARIABLE

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§1. Introduction

Throughout, ε is any small positive number, θ any real number, n, n_j, k, N some positive integers and p, p_j any primes. By $\|\theta\|$ we mean the distance from θ to the nearest integer. Write $C(\varepsilon)$, $C(\varepsilon, k)$ for positive constants which may depend on the quantities indicated inside the parentheses.

Dirichlet's theorem says that for any θ, N there exists n such that

$$n \leqslant N \quad \text{and} \quad \|\theta n\| < N^{-1}. \tag{1.1}$$

Furthermore, as a direct consequence of (1.1), there are infinitely many n such that

$$\|\theta n\| < n^{-1}. \tag{1.2}$$

Improving an estimate of Vinogradov [12], Heilbronn [6] extended (1.1) by showing that for any θ, ε, N there are n and $C(\varepsilon)$ such that

$$n \leqslant N \quad \text{and} \quad \|\theta n^2\| < C(\varepsilon) N^{-1/2+\varepsilon}. \tag{1.3}$$

Later, Davenport [3] extended (1.3) by proving that if g is a polynomial of degree $k \geqslant 2$ with real coefficients and without constant term then for any ε, N there are n and $C(\varepsilon, k)$ such that

$$n \leqslant N \quad \text{and} \quad \|g(n)\| < C(\varepsilon, k) N^{-1/(2^k-1)+\varepsilon}. \tag{1.4}$$

The results of Heilbronn [6] and Davenport [3] sparked off a series of investigations (see [9]). In particular, recently Schmidt has made remarkable progress in [9, 10]. However all these developments concerning (1.1) have no parallel results for prime. This can be seen from the following example. Let q be any positive integer having at least two

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